**Problem 1**

Write NEGATION of the following statements (**Careful, NEGATION is not the same as INVERSE**)

1. If P is a square, then P is a rectangle.
2. If n is prime, then n is odd or n is 2.
3. If Aangloo is Meena's father, then Baangloo is her uncle and Bingli is her Aunt.
4. A positive integer is prime only if it has no divisors other than 1 and itself.
5. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

**Negation:**

1. is a square and it is not a rectangle.
2. is prime and is not odd and is not 2.
3. Angloo is Meena’s father and Bangloo is not her uncle OR Bingli is not her aunt.
4. A positive integer is prime and it has divisor other 1 or itself.
5. A number is divisible by 9 and it is not divisible by 3.

**Problem 2**

Part 1: Write CONVERSE of the following statements.

1. If P is a square, then P is a rectangle.
2. If n is prime, then n is odd or n is 2.
3. If Aangloo is Meena's father, then Baangloo is her uncle and Bingli is her Aunt.
4. A positive integer is prime only if it has no divisors other than 1 and itself.
5. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

**Converse:**

1. If is a rectangle, then is a square.
2. If is odd or is 2, then is prime.
3. If Baangloo is her uncle and Bingli is her Aunt, then Aangloo is Meena's father.
4. If a number has no divisor other than 1 and itself, then it is a prime number.
5. Being divisible by 9 is a necessary condition for the number to be divisible by 3

(or we may write… If a number is divisible by 3, then it is divisible by 9)

Part 2: Write CONTRPOSITIVE of the following statements.

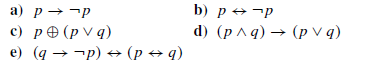
1. If P is a square, then P is a rectangle.
2. If n is prime, then n is odd or n is 2.
3. If Aangloo is Meena's father, then Baangloo is her uncle and Bingli is her Aunt.
4. A positive integer is prime only if it has no divisors other than 1 and itself.
5. Being divisible by 3 is a necessary condition for this number to be divisible by 9.

**Contrapositive:**

1. If is not a rectangle, then is not a square.
2. If is not odd and is not 2, then is not prime.
3. If Baangloo is nor her uncle OR Bingli is not her Aunt, then Aangloo is not Meena's father.
4. If a number has divisor other than 1 or itself, then it is a not prime number.
5. Being not divisible by 9 is a necessary condition for the number to be not divisible by 3.

(or we may write… If a number is not divisible by 3, then it is not divisible by 9)

Part 3: Construct a truth table for each of these compound propositions.



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|  |  |  |
| **0** | **1** | **1** |
| **1** | **0** | **0** |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| **0** | **1** | **0** |
| **1** | **0** | **0** |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| **0** | **0** | **0** | **0** |
| **0** | **1** | **1** | **1** |
| **1** | **0** | **1** | **0** |
| **1** | **1** | **1** | **0** |

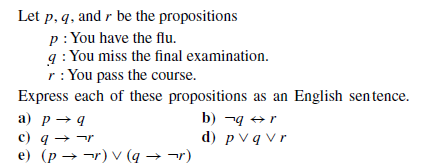


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|  |  |  |  |  |
| **0** | **0** | **0** | **0** | **1** |
| **0** | **1** | **0** | **1** | **1** |
| **1** | **0** | **0** | **1** | **1** |
| **1** | **1** | **1** | **1** | **1** |



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| **0** | **0** | **1** | **1** | **1** | **1** |
| **0** | **1** | **1** | **1** | **0** | **0** |
| **1** | **0** | **0** | **1** | **0** | **0** |
| **1** | **1** | **0** | **0** | **1** | **0** |

**Problem 3**



1. **If you have flu then you you miss the final exam.**
2. **You do not miss the final exam if and only if you pass the course.**
3. **If you miss the final exam then you do not pass the course.**
4. **You have the flu or you miss the final exam or you pass the course.**
5. **If you have the flu then you do not pass the course or if you miss the final exam then you do not pass the course.**

**Problem 4**

Part 1: Express the following statements using quantifiers, variables, and the predicates, with following information:

is set of all students

is a math major student

is a computer science major student

is an engineering major student

1. There is an engineering student who is a math major.

such that

1. Every computer science student is an engineering student.
2. No computer science students are engineering student.

such that

OR such that

1. Some computer science students are also math majors.

such that

1. Some computer science students are engineering students and some are not.

such that C.

OR such that C

Part 2: Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

**a)**

There is a number if multiplied to any other number, we get the other number back.

**In other words:** there exist multiplicative identity.

(Such integer is 1)

**b)**

Product of any two negative integers is positive

**c)**

Some numbers are less than certain number, but their square is greater than that particular number.

**OR in other words:**

There are such pair of numbers that square of one number is greater than the other, however the number is less than the other.

(negative numbers are such numbers)

**d)**

Sum of any two real number is a real number

(closure property of real number set)

Part 3: Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

**a)**

**b)**

**c)**

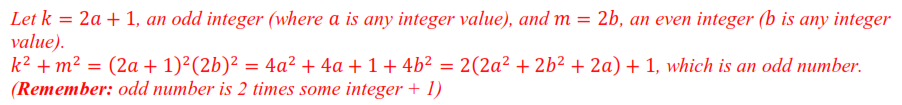
**d)**

**e)**

**Problem 5**

Part 1: **Give a direct proof of the following:**

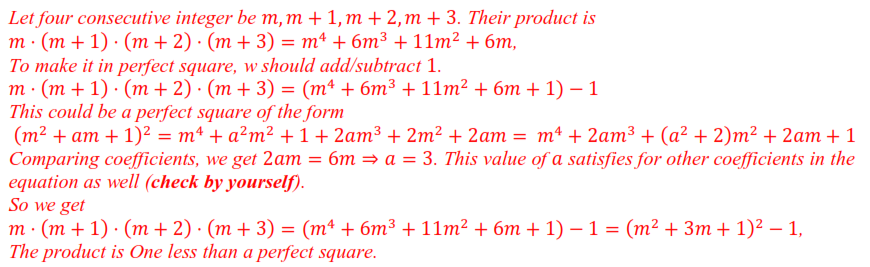
1. If is an odd integer and is even, then is odd.



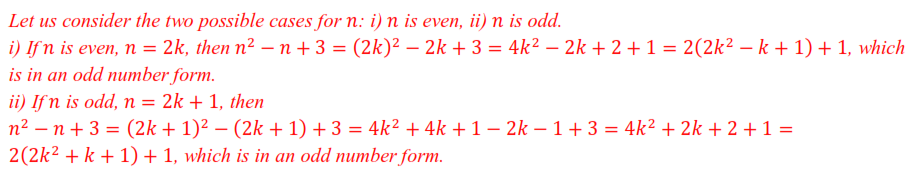
1. For any integer , is not a prime.



1. Any product of four consecutive integers is one less than a perfect square.(Hint: Take four consecutive integers as )



1. is odd. (**Hint:** *consider 2 cases: i) if is even, ii) if is odd, and then chenck whether the given expression is even or odd*)



Part 2: **Give counter-examples of the following claims to dis-prove these:**

1. For all integers , is a prime number.



1. For any integer , is prime.

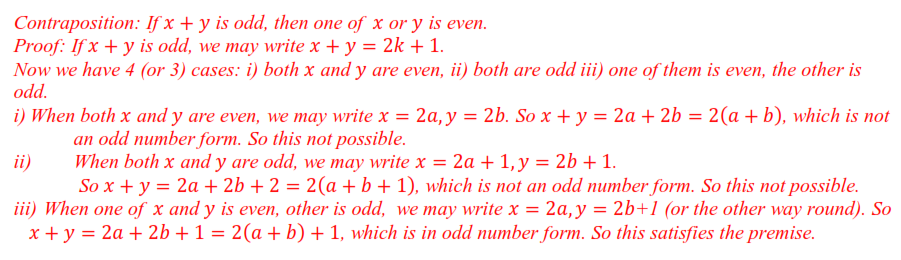


1. For all prime numbers , is prime number. (*Such prime numbers are called Mersenne primes*)

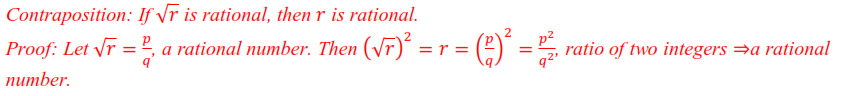


Part 3: **Prove the following using contraposition:**

1. If and are both odd, then is even. (**Be careful:** *You are asked to prove by Contraposition, although Direct Proof is easier.*)



1. If is irrational, then is irrational.



1. If is a multiple of 3, then is a multiple of 3.

